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BY S. N. SAMADDAR

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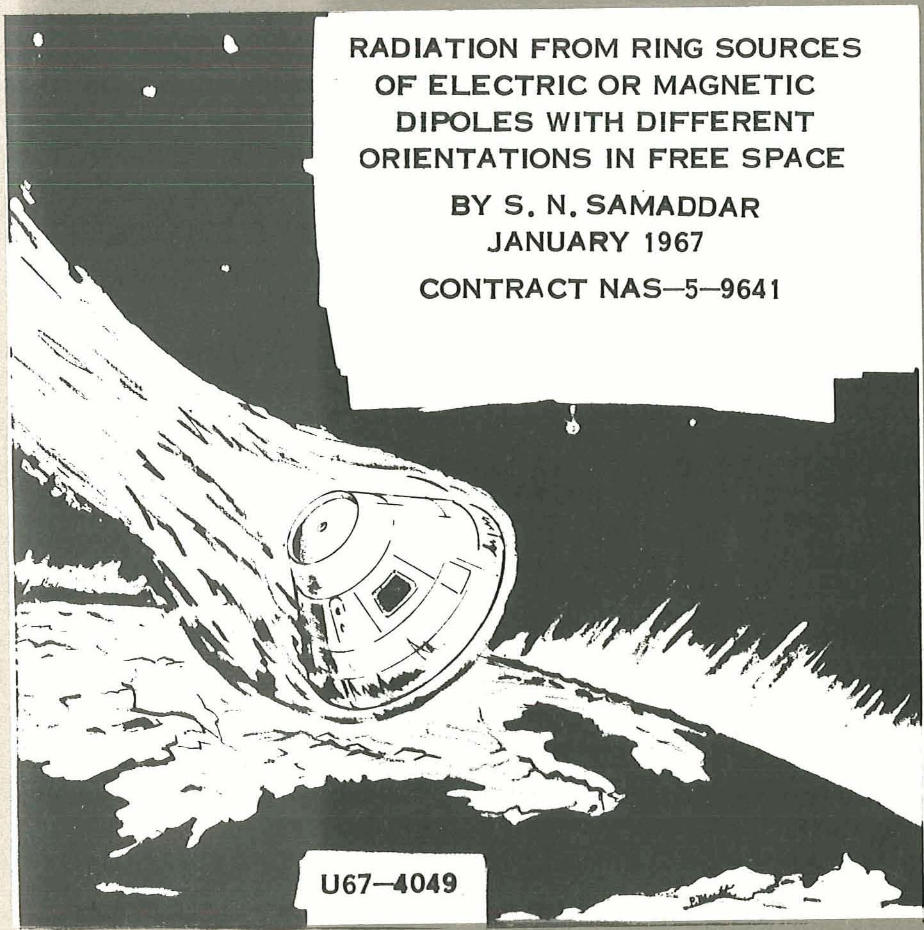

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RADIATION FROM RING SOURCES OF ELECTRIC OR MAGNETIC DIPOLES WITH DIFFERENT ORIENTATIONS IN FREE SPACE

By

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PROJECT APOLLO

RADIO BLACKOUT INVESTIGATION

Contract NAS-5-9641

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ABSTRACT

Radiation from a horizontal ring of radial electric dipoles whose strength varies as exp (inΦ) is calculated for an unbounded medium by the saddle point method of approximation. The cylindrical components of the radiated fields due to ring sources composed of radial, angular and vertical electric or magnetic dipoles, the amplitude of which varies as cos nΦ (or sin nΦ), are presented in a tabular form. The analysis was performed* as a necessary preliminary to the analysis of the effects of the plasma sheath on the Unified S-band System Antenna radiation patterns and impedance.

* Under Contract NAS5-9641 to the NASA Goddard Space Flight Center.

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SECTION 1

INTRODUCTION

This report describes an analytical study of the radiated field mode structure of rings of dipoles. The study was performed as a necessary preliminary to the analysis of the effects of the re-entry plasma sheath on the radiation patterns, and impedance of the Apollo Unified S-band System Antenna.

On the basis of the analysis of Reference 2, it was thought that an ideal loop antenna whose axis is perpendicular to the APOLLO vehicle surface and carrying a current which varies as sin ϕ or cos ϕ will radiate the same type of fields as an APOLLO Unified S-band System antenna, which is designed to launch a TE_{11} mode. However, when this loop antenna was chosen as the source of radiation in a plasma environment, it was found that there are some basic inconsistencies between this plasma problem* and the work of Reference 2. Therefore, in order to find the basic difference, the following analysis which corresponds to a simpler configuration was carried out. This shows that the analysis performed in Reference 2 is incorrect.

The general importance of gaining knowledge of the radiated fields from ring sources composed of uniform or non-uniform electric dipoles oriented in the radial, angular and vertical directions respectively of a cylindrical coordinate system (r, ϕ, z) has already been recognized.[1,2] However, the rectangular components of the radiated fields given by Knudsen[2] seem to be in error. Therefore, the purpose of this report is to outline a method of evaluation of the far field of a ring of radial electric dipoles which vary as exp ($in\phi$) in an unbounded medium and then present in a tabular form the cylindrical components of the radiated fields due to ring sources (Figure 1) consisting of radial, angular and vertical

* This analysis involving radiation through the plasma sheath will be presented in a separate report.

(z-directed) electric or magnetic dipoles, the amplitude of which varies as cos nϕ (or sin nϕ). The fields of the magnetic dipoles can also be obtained from the fields produced by electric dipoles by employing duality relations.

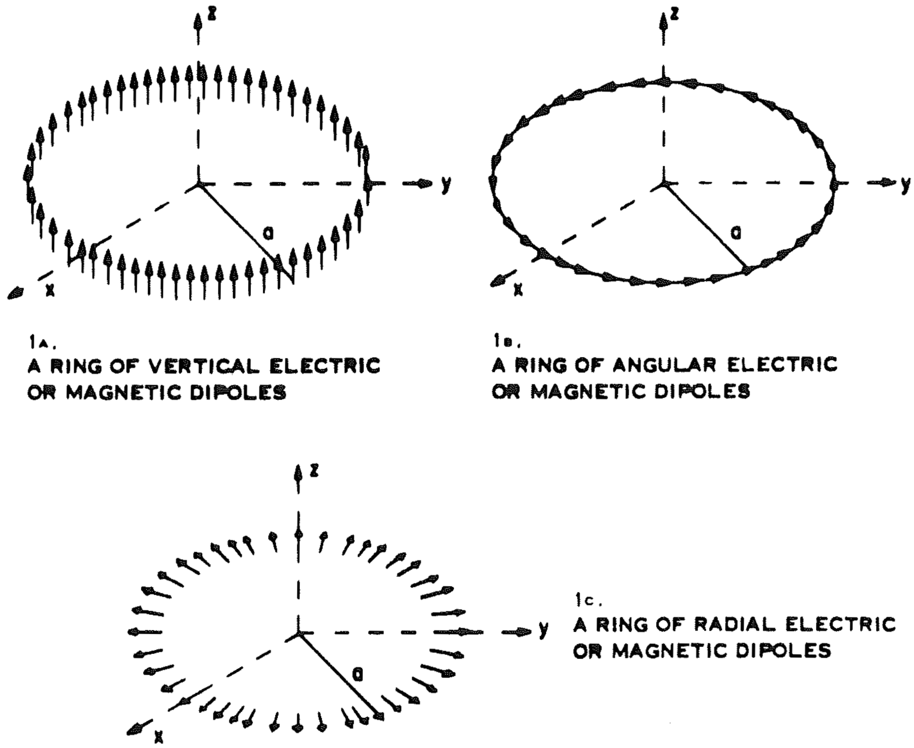

Figure 1 Ring Source Configuration

SECTION 2

ANALYTICAL PROCEDURE

For the present investigation, we express Maxwell's equations with electric, $\underline{J}$, and magnetic, $\underline{M}$, sources in the following manner in cylindrical coordinates [suppressing the assumed time dependence exp $(-i\omega t)$]:

$$\left(K^2 + \frac{\partial^2}{\partial z^2}\right) \underline{E}_t = \frac{\partial}{\partial z} \nabla_t E_z + i\omega\mu \nabla_t H_z \times \underline{Z}_o - i\omega\mu \underline{J}_t - \frac{\partial}{\partial z} \underline{M}_t \times \hat{\underline{Z}}_o \tag{1}$$

$$\left(K^2 + \frac{\partial^2}{\partial z^2}\right) \underline{H}_t = \frac{\partial}{\partial z} \nabla_t H_z + i\omega\epsilon \hat{\underline{Z}}_o \times \nabla_t E_z - i\omega\epsilon \underline{M}_t - \frac{\partial}{\partial z} \hat{\underline{Z}}_o \times \underline{J}_t \tag{2}$$

$$(\nabla^2 + K^2) E_z = -i\omega\mu J_z + \frac{1}{i\omega\epsilon} \frac{\partial}{\partial z} \nabla \cdot \underline{J} + \frac{1}{r}\frac{\partial}{\partial r} (rM_\phi) - \frac{1}{r}\frac{\partial}{\partial \phi} M_r \tag{3}$$

$$(\nabla^2 + K^2) H_z = -i\omega\epsilon M_z + \frac{1}{i\omega\mu} \frac{\partial}{\partial z} \nabla \cdot \underline{M} - \frac{1}{r}\frac{\partial}{\partial r} (rJ_\phi) + \frac{1}{r}\frac{\partial}{\partial \phi} J_r \tag{4}$$

$$K^2 = \omega^2 \mu\epsilon$$

where for any vector $\underline{P} = \underline{P}_t + \hat{\underline{Z}}_o P_z$ and $\nabla = \nabla_t + \hat{\underline{Z}}_o \frac{\partial}{\partial z}$; $\hat{\underline{Z}}_o$ being the unit vector in the z-direction.

The relations (1) to (4) suggest that if the sources depend on all the coordinates r, ϕ, z, then J_r, J_ϕ, M_r and M_ϕ will excite all the components of the $\underline{E}$ and $\underline{H}$, whereas J_z and M_z will not generate the fields H_z and E_z respectively in an unbounded region.

The ring sources $\underline{J}$ and $\underline{M}$ can be represented in the following manner;

$$\underline{J} = \hat{\underline{i}}_e \frac{\mathcal{J} f(\phi)\ \delta(r-a)\ \delta(z)}{2\pi r}, \qquad \underline{M} = \hat{\underline{i}}_m \frac{m f(\phi)\ \delta(r-a)\ \delta(z)}{2\pi r} \tag{5}$$

where a is the radius of the ring, $\hat{\underline{i}}_e$ and $\hat{\underline{i}}_m$ are the unit vectors in the direction of electric and magnetic dipoles respectively. The quantitites $\mathcal{J}$ and m are the constant amplitudes of $\underline{J}$ and $\underline{M}$ respectively. The function $f(\phi)$ may be one of exp $(in\phi)$, cos $n\phi$ and sin $n\phi$, n being an integer. For a ring of radial electric dipoles, we assume $f(\phi) = \exp(in\phi)$, $E_z = e^{in\phi}\, \psi_{1n}(r,z)$ and $H_z = ie^{in\phi}\, \psi_{2n}(r,z)$. If $S(r,z)$ is any well behaved function of r and z, we introduce its Fourier transform in the following manner;

$$\hat{S}(r,\zeta) = \int_{-\infty}^{\infty} S(r,z)\, e^{-i\zeta z} dz, \qquad S(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{S}(r,\zeta)\, e^{i\zeta z} d\zeta \tag{6}$$

If $\hat{\psi}_n(r,\zeta)$ is the Fourier transform of $\psi_n(r,z)$, then it can be shown that,

$$\hat{\psi}_{1n}(r,\zeta) = \frac{\zeta}{\omega\epsilon}\, \frac{\partial}{\partial a}\, \hat{\psi}_n, \qquad \hat{\psi}_{2n}(r,\zeta) = -\frac{n}{a}\, \hat{\psi}_n \tag{7}$$

and where $\hat{\psi}_n$ satisfies the following inhomogeneous differential equation;

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\hat{\psi}_n\right) + (\eta^2 - n^2/r^2)\, \hat{\psi}_n = -\frac{\mathcal{J}\delta(r-a)}{2\pi r}, \quad \eta^2 = K^2 - \zeta^2 \tag{8}$$

Therefore, $\hat{\psi}_n$ can be expressed as,

$$\hat{\psi}_n = i\mathcal{J}\, J_n(\eta r_<)\, H_n^{(1)}(\eta r_>)/4 \tag{9}$$

where for $r < a$, $r_< = r$, $r_> = a$ and for $r > a$, $r_< = a$, $r_> = r$.

Now, for $r > a$, E_z and H_z can be expressed in the following manner by employing inverse Fourier transform together with the use of (9) and (8);

$$E_z = \frac{i\mathcal{J} e^{in\phi}}{8\pi\omega\epsilon}\int_{-\infty}^{\infty} \zeta\eta\, J_n'(\eta a)\, H_n^{(1)}(\eta r) e^{i\zeta z} d\zeta, \quad r > a \tag{10}$$

and

$$H_z = \frac{\gamma n e^{in\phi}}{8\pi a} \int_{-\infty}^{\infty} J_n(\eta a)\, H_n^{(1)}(\eta r)\, e^{i\zeta z}\, d\zeta, \quad r > a \tag{11}$$

Using the standard procedure[3], the asymptotic values of (10) and (11) for $|\eta r| >> 1$ can be given by (with $r = R \sin\theta$, $z = R\cos\theta$)

$$E_z \sim \frac{\gamma \omega \mu}{2}\, e^{in\phi} \sin 2\theta\, Q_n; \quad \text{and } H_z \sim \frac{-in\gamma}{a}\, e^{in\phi}\, F_n \tag{12}$$

where,

$$F_n = \frac{e^{iKR - in\pi/2}}{4\pi R} J_n(Ka \sin\theta) \quad \text{and} \quad Q_n = \frac{e^{iKR - in\pi/2}}{4\pi R} J_n'(Ka \sin\theta) \tag{13}$$

All other components can be calculated by using (1), (2), (12) and (13).

SECTION 3

RESULTS

Following the above procedure, all the cylindrical components of the radiated electromagnetic fields due to various ring sources, the amplitudes of which vary as cos $n\phi$, were calculated and are presented in Table 1. To obtain results for ring sources which vary as sin $n\phi$ from the corresponding tabulated results, one has to change cos $n\phi$ to sin $n\phi$ and sin $n\phi$ to $-$cos $n\phi$. Thus, knowing the results for both the angular variations, cos $n\phi$ and sin $n\phi$ of the ring source, one may easily calculate the radiated fields due to a ring source of arbitrary angular variations which can be expressed as the linear superposition of cos $n\phi$ and sin $n\phi$ (including the angular variation exp $(in\phi)$). A close inspection of the tabulated results shows that though a uniform (i.e., $n = 0$) electric current in a loop (or ring) may be represented by a magnetic dipole, a non-uniform ($n \neq 0$) electric current in a loop cannot be represented by a magnetic dipole alone. This statement can also be justified through an appropriate display of Maxwell's equations with sources as shown in (1) to (4).

The rectangular and spherical components of the fields can also be obtained from the cylindrical components by using the following relations:

$$A_x = A_r \cos\phi - A_\phi \sin\phi; \quad A_y = A_r \sin\phi + A_\phi \cos\phi; \quad A_z = A_z \tag{14a}$$

$$A_R = A_r \sin\theta + A_z \cos\theta; \quad A_\theta = A_r \cos\theta - A_z \sin\theta; \quad A_\phi = A_\phi \tag{14b}$$

where the vector $\underline{A}$ represents either $\underline{E}$ or $\underline{H}$.

TABLE 1

CYLINDRICAL COMPONENTS OF THE RADIATED FIELD PRODUCED BY RING SOURCES WHICH VARY AS COS Nϕ IN FREE SPACE

Component of Ring Source Which Varies as cos nϕ	E_r	E_ϕ	E_z	H_r	H_ϕ	H_z
J_r	$-\mathcal{J}\omega\mu \cos n\phi \cos^2\theta\, Q_n$	$\frac{\mathcal{J} n\omega\mu}{Ka \sin\theta} \sin n\phi\, F_n$	$\frac{\mathcal{J}\omega\mu}{2} \cos n\phi \sin 2\theta\, Q_n$	$-\frac{\mathcal{J} n}{a} \sin n\phi \cot\theta\, F_n$	$-\mathcal{J}K \cos n\phi \cos\theta\, Q_n$	$\frac{\mathcal{J} n}{a} \sin n\phi\, F_n$
J_ϕ	$\frac{-\mathcal{J} n\omega\mu}{Ka \sin\theta} \sin n\phi \cos^2\theta\, F_n$	$-\mathcal{J}\omega\mu \cos n\phi\, Q_n$	$\frac{\mathcal{J} nK}{\omega\epsilon a} \sin n\phi \cos\theta\, F_n$	$\mathcal{J}K \cos n\phi \cos\theta\, Q_n$	$-\frac{\mathcal{J} n}{a} \sin n\phi \cot\theta\, F_n$	$-\mathcal{J}K \cos n\phi \sin\theta\, Q_n$
J_z	$\frac{-i\mathcal{J}\omega\mu}{2} \cos n\phi \sin 2\theta\, F_n$	$O(1/R^2)$	$i\mathcal{J}\omega\mu \cos n\phi \sin^2\theta\, F_n$	$O(1/R^2)$	$-i\mathcal{J}K \cos n\phi \sin\theta\, F_n$	0
M_r	$\frac{mn}{a} \sin n\phi \cot\theta\, F_n$	$mK \cos n\phi \cos\theta\, Q_n$	$-\frac{mn}{a} \sin n\phi\, F_n$	$-m\omega\epsilon \cos n\phi \cos^2\theta\, Q_n$	$\frac{m n\omega\epsilon}{Ka \sin\theta} \sin n\phi\, F_n$	$\frac{m\omega\epsilon}{2} \cos n\phi \sin 2\theta\, Q_n$
M_ϕ	$-mK \cos n\phi \cos\theta\, Q_n$	$\frac{mn}{a} \sin n\phi \cot\theta\, Q_n$	$mK \cos n\phi \sin\theta\, Q_n$	$-\frac{mn\omega\epsilon}{Ka \sin\theta} \sin n\phi \cos^2\theta\, F_n$	$-m\omega\epsilon \cos n\phi\, Q_n$	$\frac{mnK}{\omega\mu a} \sin n\phi \cos\theta\, F_n$
M_z	$O(1/R^2)$	$imK \cos n\phi \sin\theta\, F_n$	0	$-\frac{im\omega\epsilon}{2} \cos n\phi \sin 2\theta\, F_n$	$O(1/R^2)$	$im\omega\epsilon \cos n\phi \sin^2\theta\, F_n$

SECTION 4

REFERENCES

1. Moullin, E.B., "Radiation from Large Circular Loops," Jour. IEE, (London), Part III, 1946, 93, p345.

2. Knudsen, H.L., "The Field Radiated by a Ring Quasi-Array of an Infinite Number of Tangential or Radial Dipoles," Proc. IRE, 1953, 41, p781.

3. Brekhovskikh, L.M., "Waves in Layered Media", Academic Press, 1960, p245.

SECTION 5

LIST OF SYMBOLS

$\underline{E}_t$ = Transverse (x and y) part of the electric field vector, of which the longitudinal part is E_z (z-component)

$\underline{H}_t$ = Transverse part of the magnetic field vector

$\underline{M}_t$ = Transverse part of the magnetic current vector, $\underline{M}$

$\underline{J}_t$ = Transverse part of the electric current vector, $\underline{J}$

$\frac{\partial}{\partial z}$ = First derivative with respect to the coordinate z

$\underline{Z}_o$ = Unit vector in the z - direction

K^2 = $\omega^2 \mu_o \varepsilon_o$

ω = Angular frequency of the antenna

μ_o = Permeability of vacuum

ε_o = Permittivity of vacuum

E_z, H_z = z-component of the vectors $\underline{E}$ and $\underline{H}$ respectively

∇^2 = Laplacian operator

∇_t = $\nabla - \hat{\underline{z}}_o \frac{\partial}{\partial z}$

$\underline{A}$ and $\underline{P}$ = arbitrary vectors

f (ϕ) = A function of ϕ

Subscripts r, ϕ, z attached to a quantity represent r, ϕ and z cylindrical components respectively of a vector.

$\psi_{1n}(r, z)$ = $e^{-in\phi} E_z$

$\psi_{2n}(r, z)$ = $-ie^{-in\phi} H_z$

$\psi_n(r, z)$, $S(r, z)$ are scalar functions of r and z.

$\hat{S}(r, \zeta)$, $\hat{\psi}_{1n}(r, \zeta)$, $\hat{\psi}_{2n}(r, \zeta)$ and $\hat{\psi}_n(r, \zeta)$ are Fourier transform of $S(r, z)$, $\psi_{1n}(r, z)$, $\psi_{2n}(r, z)$ and $\psi_n(r, z)$ respectively with respect to $e^{-i\zeta z}$.

ζ = Transform variable of Fourier Transform

η^2 = $K^2 - \zeta^2$

$\delta(r - a)$, $\delta(z)$ = Dirac's delta functions

a = radius of the ring soucres

$\hat{\underline{i}}_e$ = Unit vector in the direction of the electric current

$\hat{\underline{i}}_m$ = Unit vector in the direction of the magnetic current

$\mathcal{J}$ = Strength or amplitude of the electric current $\underline{J}$

m = Strength or amplitude of the magnetic current $\underline{M}$

F_n = $e^{ikR - in\pi/2} \cdot J_n(ka \sin\theta) / 4\pi R$

Q_n = $e^{ikR - in\pi/2} \cdot J_n'(ka \sin\theta) / 4\pi R$

R = $\sqrt{r^2 + z^2}$, or $R = r/\sin\theta = z/\cos\theta$

$J_n(ka \sin\theta)$ = Bessel Function of the first kind and order n with the argument $ka \sin\theta$

$J_n'(ka \sin\theta)$ = Derivative of $J_n(ka \sin\theta)$ with respect to its argument